Unit
03

THE NUMBER SYSTEMS

UNIT STRUCTURE

3.0 Learning Objectives :

After working through this unit, you should be able to :

- Understand different number systems
- How data will be represented in the computer memory
- List different types of Number Systems
- Convert a number from one number system to another

3.1 Introduction :

As we have discussed that the computer is an electronic device, which made by using Integrated Circuits (ICs). ICs itself made by millions of small electronic components like capacitors, transistors etc. These devices can be in charge or discharge state. That means they can store only 1 or 0.

That is the reason we have written that the machine can understand only Machine or Binary language. It is a language of two symbols those are 1 and 0.

We have also discussed that the data given by the user as an input will be placed on main memory of the system called Random Access Memory (RAM). RAM is also made by semi–conductor material. Which means whatever data or information we are storing in the main memory will be stored in the form of Binary. In this chapter we will discuss how can compute binary, not only that apart from binary how many different number systems are available.

3.2 Decimal Number System :

There are four number systems are available :

1. Binary Number System
2. Octal Number System
3. Decimal Number System
4. Hexa–Decimal Number System

In our day to day life we use the decimal number system. Decimal number system uses 10 different symbols to represent the number, those symbols are {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. Because of there are 10 different symbols are used in the decimal number system, the base of decimal number system is 10. Number 5,123 means $5 * 10^3 + 1 * 10^2 + 2 * 10^1 + 3 * 10^0 = 5 * 1000 + 1 * 100 + 2 *10 + 3 * 1 = 5{,}000 + 100 + 20 + 3 = 5{,}123$.

3.3 Binary Number System :

Binary is the number system, which use only two symbols to represent the number, those are {0, 1}. Therefore, the base of the Binary number system is 2. Consider the following table. In the table, we have given Binary of first 20 decimal numbers.

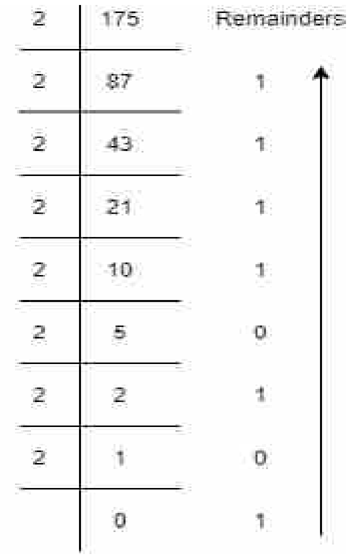
Decimal	Binary		Decimal	Binary
0	0		11	1011
1	1		12	1100
2	10		13	1101
3	11		14	1110
4	100		15	1111
5	101		16	10000
6	110		17	10001
7	111		18	10010
8	1000		19	10011
9	1001		20	10100
10	1010			

3.3.1 Conversion between Decimal and Binary :

To convert any Decimal number into Binary, you need to divide that number by 2 (as the base of Binary number system is 2). After division you have to record quotient and remainder. Again, the quotient part you need to divide by 2 and you need to record further quotient and remainder. This process has to be continued till the number does not becomes 0. Once the number turns to 0, you need to write all remainders in the reverse order and you will get the binary of that number. See the example below in which we shown how to find a binary of decimal number 175).

Therefore, the binary of the decimal number 175 is 1010 1111, which can be written as :

$$(175)_{10} = (1010\ 1111)_2$$



2	175	Remainders
2	87	1
2	43	1
2	21	1
2	10	1
2	5	0
2	2	1
2	1	0
	0	1

In the previous example we have seen that how can we convert the decimal number into its equivalent binary. We have seen that the binary of decimal number 175 is 1010 1111. Now the question is can we convert the binary to its equivalent decimal ? Let if we have binary '1010 1111', then can we obtain decimal 175 from it ? The answer is 'YES'. See the example given below in which we have explained how a binary number can be converted into its equivalent decimal.

1	0	1	0	1	1	1	1
$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
128	0	32	0	8	4	2	1

To obtain decimal number from the binary number, each binary digit will be multiplied by 2^0, 2^1, 2^2, … 2^{n-1} from right to left (Where n = total binary digits count). That means each binary digit will be multiplied by 1, 2, 4, 8, 16… as so on. Finally, we need to add all numbers (128 + 0 + 32 + 0 + 8 + 4 + 2 + 1) and we will get its equivalent decimal number that is 175.

❑ **Check Your Progress – 1 :**

1. $(395)_{10} = (\_\_\_\_\_\_)_2$

   [A] 10011011 [B] 110100011 [C] 110001011 [D] 110110010

2. $(434)_{10} = (\_\_\_\_\_\_)_2$

   [A] 10011011 [B] 110100011 [C] 110001011 [D] 110110010

3. $(10011011)_2 = (\_\_\_\_\_\_)_{10}$

   [A] 155 [B] 419 [C] 395 [D] 434

4. $(110100011)_2 = (\_\_\_\_\_\_)_{10}$

   [A] 155 [B] 419 [C] 395 [D] 434

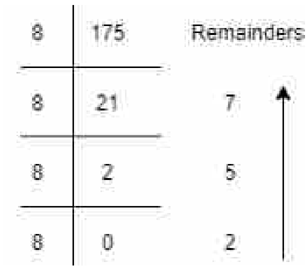
3.4 Octal Number System :

Octal number system uses 8 symbols to represent any number, those are {0, 1, 2, 3, 4, 5, 6, 7}. Therefore, the base of octal number system is 8. In octal number system we are not using symbols 8 and 9. For example, 129 is not a valid octal number as it has one digit 9. The following table represents Decimal, Octal and Binary numbers from 0 to 21.

Decimal	Octal	Binary	Decimal	Octal	Binary
0	0	0	11	13	1011
1	1	1	12	14	1100
2	2	10	13	15	1101
3	3	11	14	16	1110
4	4	100	15	17	1111
5	5	101	16	20	10000
6	6	110	17	21	10001
7	7	111	18	22	10010
8	10	1000	19	23	10011
9	11	1001	20	24	10100
10	12	1010	21	25	10101

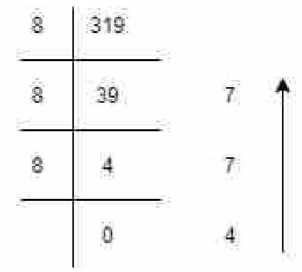
3.4.1 Conversion between Octal and Decimal :

To convert any given decimal number into its equivalent octal number, you need to divide that number by 8 till the number does not becomes 0. Each time you have to note down the remainder. Finally, write down all remainder into reverse order. For example, if we want to convert octal of decimal number 175 then,



Therefore, the octal of the decimal number 175 is 257, which can be written as :

$$\mathbf{(175)_{10} = (257)_8}$$



The octal of the decimal number 319 is 477, which can be written as :

$$\mathbf{(319)_{10} = (477)_8}$$

To convert any octal number to decimal number, we need to multiply each digit of an octal by (8^0, 8^1, 8^2, … 8^{n-1}) from right to left (where n is a count of number of digits in an octal number). That means, each digit of an octal number has to be multiplied by (1, 8, 64 … and so on) from right to left. Finally, we need to sum all multiples to obtain its equivalent decimal.

For Example, if we want to compute decimal of octal number 257, then

2	**5**	**7**
8^2 = **64**	8^1 = **8**	8^0 = **1**
128	**40**	**7**

2 * 84 = 128, 5 * 8 = 40 and 7 * 1 = 7. Sum of 128 + 40 + 7 = 175. That means $(257)_8 = (175)_{10}$.

Similarly, octal number 477 can be converted into its equivalent decimal as follows :

4	7	7
8^2 = **64**	8^1 = **8**	8^0 = **1**
256	**56**	**7**

$(477)_8 = (4 * 8^2 + 7 * 8^1 + 7 * 8^0)_{10}$

$= (4 * 64 + 7 * 8 + 7 * 1)_{10}$

$= (256 + 56 + 7)_{10}$

$= (319)_{10}$

❑ **Check Your Progress – 2 :**

1. $(395)_{10} = (\_\_\_\_\_\_)_8$

   [A] 316 [B] 266 [C] 613 [D] 662

2. $(434)_{10} = (\_\_\_\_\_\_)_8$

   [A] 316 [B] 266 [C] 613 [D] 662

3. $(763)_8 = (\_\_\_\_\_\_)_{10}$

   [A] 489 [B] 499 [C] 419 [D] 434

4. $(643)_8 = (\_\_\_\_\_\_)_{10}$

   [A] 419 [B] 499 [C] 395 [D] 434

3.4.2 Conversion between Octal and Binary :

One obvious method to convert any octal number into the binary is : [1] Convert the given octal number into decimal and, [2] Convert that decimal number into binary. For Example : $(257)_8$ will be converted first into $(175)_{10}$ and then $(175)_{10}$ will be converted into $(1010\ 1111)_2$.

But is there any easier method is there ? Obviously, yes. To convert any octal number into binary, or to convert any binary number to octal memorise 3–bits binary of all octal symbols. Now one question will come in your mind that why 3–bits binary of all octal symbols ? The answer is, in octal number system, we have 8 different symbols, and in binary we have 2 symbols. To represent 8 different octal symbols, we need 3–bits (as $2^3 = 8$).

Octal	Binary		Octal	Binary
0	000		4	100
1	001		5	101
2	010		6	110
3	011		7	111

To convert any octal number directly into the binary you need to write 3–bits of binary for each of its octal digit :

For Example, if we want to convert $(257)_8$ into the binary then :

Binary of 2 = 010, Binary of 5 = 101 and Binary of 7 = 111. Mixed up all binaries and remove all zeros from the left hand–side.

$(257)_8 = (\underline{010}\ \underline{101}\ \underline{111}) = 010101111 = (10101111)_2$ [Remove all zeros from the left side].

Similarly, to convert any binary number to octal you need make pair of 3–bits. If total number of bits are not in multiple of 3 then you can append 1 to 2 zeros at the left side. Now write single digit octal code for each 3–bits binary from the table given above.

For Example, to convert $(10011011)_2$ into octal, we need to make pair of 3–bits from right to left that is : 10 <u>011</u> <u>011</u>. Now to complete 3–bits binary of 10 append one zero to left side. So, after adding one zero to the left side we have : <u>010</u> <u>011</u> <u>011</u>.

Now from the table we can see that 010 is a binary of : 2, 011 is a binary of : 3 and 011 is a binary of : 3. Therefore, $(10011011)_2 = (233)_8$.

❑ **Check Your Progress – 3 :**

1. $(613)_8$ = (________)$_2$

   [A] 110001011 [B] 011001101 [C] 110001110 [D] 111000101

2. $(5371)_8$ = (________)$_2$

   [A] 101011111001 [B] 110011111001

   [C] 101110111011 [D] 001111011101

3. $(101011111001)_2$ = (________)$_8$

   [A] 5471 [B] 5371 [C] 5741 [D] 5714

4. $(11001101110)_2$ = (________)$_8$

   [A] 3516 [B] 1356 [C] 6334 [D] 3156

3.5 Hexa–Decimal Number System :

Hexa–decimal number system uses 16 different symbols, those are {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}. Refer the following table in which we have given Decimal, Hexa–decimal, Octal and Binary from 1 to 32.

D	H	O	B	D	H	O	B	D	H	O	B
0	0	0	0	**11**	B	13	1011	**22**	16	26	10110
1	1	1	1	**12**	C	14	1100	**23**	17	27	10111
2	2	2	10	**13**	D	15	1101	**24**	18	30	11000
3	3	3	11	**14**	E	16	1110	**25**	19	31	11001
4	4	4	100	**15**	F	17	1111	**26**	1A	32	11010
5	5	5	101	**16**	10	20	10000	**27**	1B	33	11011
6	6	6	110	**17**	11	21	10001	**28**	1C	34	11100
7	7	7	111	**18**	12	22	10010	**29**	1D	35	11101
8	8	10	1000	**19**	13	23	10011	**30**	1E	36	11110
9	9	11	1001	**20**	14	24	10100	**31**	1F	37	11111
10	A	12	1010	**21**	15	25	10101	**32**	20	40	100000

In the above table D represent Decimal number, H represents Hexa–Decimal number, O represents Octal number and B represents Binary number.

3.5.1 Conversion between Hexa–Decimal and Decimal :

To convert any number from decimal to hexa–decimal, you need to divide that number by 16 (as the base of the hexa–decimal number system is 16) and you have to record remainder till that number turns to 0. Once the number becomes 0, you need to write all remainders in reverse order. Make sure if the remainder is 10 then you need to write 'A', if it is 11 then you need to write 'B' and in that way if remainder is 15 then you need to write 'F'.

For example, if you want to convert decimal 242 into hexa–decimal then,

16	242	Remainders	
16	15	2	2 ↑
	0	15	F

The hexa–decimal of given decimal 242 is F2. That can be written as :

$$(242)_{10} = (F2)_{16}$$

16	986	Remainders	
16	61	10	A ↑
16	3	13	D
	0	3	3

The hexa–decimal of given decimal 986 is 3DA. That can be written as :

$$(986)_{10} = (3DA)_{16}$$

If you want to convert any hexa–decimal number into decimal then you need to multiply each digit of hexa–decimal with 16^0, 16^1, 16^2 … 16^{n-1}, where n is the total number of digits of hexa–decimal number from right to left. That means the last digit will be multiplied with 1, second last digit will be multiplied with 16, third last digit will be multiplied with 256 and so on. If you have hexa–decimal digit 'A' then you have considered it as 10, 'B' then consider it as 11, and so on. The last hexa–decimal digit 'F' means 15.

Consider the following 2 examples :

[1] $(F2)_{16} = (F * 16^1 + 2 * 16^0)_{10}$

$= (F * 16 + 2 * 1)_{10}$

$= (15 * 16 + 2 * 1)_{10}$

$= (240 + 2)_{10}$

$= (242)_{10}$

[2] $(3DA)_{16} = (3 * 256 + A * 16 + A * 1)_{10}$

$= (3 * 256 + 13 * 16 + 10 * 1)_{10}$

$= (768 + 208 + 10)_{10}$

$= (986)_{10}$

❑ Check Your Progress – 4 :

1. $(1F9)_{16}$ = (________)$_{10}$
2. $(FF)_{16}$ = (________)$_{10}$
3. $(777)_{10}$ = (________)$_{16}$
4. $(495)_{10}$ = (________)$_{16}$

3.5.2 Conversion between Hexa–decimal and Octal :

As we know that in the base of the hexa–decimal number system is 16. To represent 16 symbols, we need 4–Bits (as 24=16). Refer the following table to convert given decimal number into hexa–decimal number.

Hexa–Decimal	Binary		Hexa–Decimal	Binary
0	0000		8	1000
1	0001		9	1001
2	0010		A	1010
3	0011		B	1011
4	0100		C	1100
5	0101		D	1101
6	0110		E	1110
7	0111		F	1111

So, if you want to convert hexa–decimal 3AD into binary then,

3	A	D	3AD
0011	1010	1101	1110101101

[Remove all zeros from the left side of the obtained binary number]

Similarly, if you want to convert any binary number into hexa–decimal number then we need to make pair of 4–Bits from the right to left. If the left most pair doesn't have 4–Bits, then add 0s at the left side to complete pair of 4. Now, put equivalent hexa–decimal digit from the table given above to make your hexa–decimal number.

For example, if we want to compute $(1010101111)_2$ into hexa–decimal then we will make pair of 4–Bits from right side.

<u>10</u> <u>1010</u> <u>1111</u>

Now to complete the pair of 4–Bits at the left most pair add two 0s to it at left side.

<u>0010</u> <u>1010</u> <u>1111</u> Now, put equivalent hexa–decimal digit from the table. From the table we can see that 0010 = 2, 1010 = A and 1111 = F.

Therefore, the hexa–decimal number of the binary number $(1010101111)_2$ is $(2AF)_{16}$.

❑ **Check Your Progress – 5 :**

1. $(1F9)_{16}$ = (________)$_2$
2. $(FF)_{16}$ = (________)$_2$
3. $(10110110101)_2$ = (________)$_{16}$
4. $(110101111001)_2$ = (________)$_{16}$

3.5.3 Conversion between Hexa–decimal and Binary :

To convert Hexa–decimal number into octal, first you convert the given hexa–decimal number into binary. Once you get the binary then you convert the binary number into octal. We have already discussed how can we convert a hexa–decimal number into binary as well as how to convert a binary number into octal in the previous sections :

Consider the following examples :

[1] $(1F9)_{16}$ = $(\_\_\_\_\_\_\_)_8$

$(1F9)_{16}$ = $(0001\ 1111\ 1001)_2$

= $(000\ 111\ 111\ 001)_2$

= $(771)_8$ [Write octal for each pair and discard first 0]

[2] $(5F)_{16}$ = $(\_\_\_\_\_\_\_)_8$

$(5F)_{16}$ = $(0101\ 1111)_2$

= $(01\ 011\ 111)_2$

= $(001\ 011\ 111)_2$

[Add one 0 to left side to complete pair of 3–Bits]

= $(1\ 3\ 7)_8$ [Write octal for each pair of 3–bits]

Similarly, if you want to convert any octal number into hexa–decimal number then first write 3–bits of binary for each octal digit, to convert given octal number into binary. Once you get the binary, make group of 4–Bits from right to left. Write hexa–decimal for each group of 4–bits and you will get hexa–decimal number.

For Example : $(137)_8 = (001\ 011\ 111)_2 = (0\ 0101\ 1111)_2 = (5F)_{16}$.

❑ **Check Your Progress – 6 :**

1. $(1D9)_{16}$ = $(\_\_\_\_\_\_\_)_8$
2. $(F3A)_{16}$ = $(\_\_\_\_\_\_\_)_8$
3. $(613)_8$ = $(\_\_\_\_\_\_\_)_{16}$
4. $(5371)_8$ = $(\_\_\_\_\_\_\_)_{16}$

3.6 Let Us Sum Up :

In this unit, we :

- Understand Decimal, Binary, Octal and Hexa–Decimal number systems.
- Have learn how to convert Decimal number to Binary and Binary number to Decimal.
- Have learn how to convert Octal numbers to Decimal and Decimal to Octal.
- Studied how to convert Octal numbers to Binary and Binary numbers to Octal
- Discussed how to convert Hexa–Decimals to Decimals and Decimal to Hexa–Decimal numbers.
- Learn how to convert Hexa–Decimal numbers to Binary and Binary numbers to Hexa–Decimal numbers.
- Studied how to convert Hexa–Decimal numbers into Octal and Octal numbers into Hexa–Decimal numbers.

3.7 Suggested Answers For Check Your Progress :

❑ **Check Your Progress 1 :**

1. [C] 2. [D] 3. [A] 4. [B]

❑ **Check Your Progress 2 :**

1. [C] 2. [D] 3. [B] 4. [A]

- **Check Your Progress 3 :**

  1. [A] 2. [A] 3. [B] 4. [D]

- **Check Your Progress 4 :**

  1. 505 2. 255 3. 309 4. 1EF

- **Check Your Progress 5 :**

  1. 1 1111 1001 2. 1111 1111 3. 5B5 4. D79

- **Check Your Progress 6 :**

  1. 731 2. 7472 3. 18B 4. AF9

3.8 Glossary :

Decimal : It is a number system with 10 symbols, those are : {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Binary : It is a number system with 2 symbols, those are {0, 1}. All electronics devices are store data and information using binary number system.

Octal : It is a number system with 8 symbols, those are : {0, 1, 2, 3, 4, 5, 6, 7}

Hexa–Decimal : It is a number system with 16 symbols, those are : {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}

3.9 Assignment :

1. What is Decimal number system ? Explain it with an example.
2. What is Binary number system ? How can we convert a decimal number to Binary ?
3. What is Hexa–Decimal number system ? List all symbols used in it.

3.10 Activity :

1. $(5723)_{10} = (\_\_\_\_\_\_)_{2} = (\_\_\_\_\_\_)_{8} = (\_\_\_\_\_\_)_{16}$
2. $(101111010)_{2} = (\_\_\_\_\_\_)_{10} = (\_\_\_\_\_\_)_{8} = (\_\_\_\_\_\_)_{16}$
3. $(3725)_{8} = (\_\_\_\_\_\_)_{2} = (\_\_\_\_\_\_)_{10} = (\_\_\_\_\_\_)_{16}$
4. $(A65D)_{16} = (\_\_\_\_\_\_)_{2} = (\_\_\_\_\_\_)_{8} = (\_\_\_\_\_\_)_{10}$

3.11 Case Study :

Find, how can we convert a decimal number with floating points into Binary. Convert (443.125)10 into its equivalent binary number.

3.12 Further Reading :

1. Computer Fundamentals by P.K.Sinha and Priti Sinha.
2. Discovering Computers 2016 by Shelly Cashman Series. CENGAGE publications.
3. Computer Fundamentals by Pearl Software, Khanna Book Publishing.